

Yang-Mills mass gap at large- N , topological quantum field theory and hyperfiniteness

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In the large- N limit of pure $SU(N)$ Yang-Mills, the ambient algebra of Wilson loops is known to be a type II_1 non-hyperfinite factor. Nevertheless, at the leading $1/N$ non-trivial order, because of the mass gap and confinement, the connected two-point correlation functions of local gauge invariant operators are conjectured to be an infinite sum of propagators of free massive fields. It is an open problem, most relevant to a complete solution of the glueball spectrum at large- N , whether or not the corresponding local algebra is hyperfinite. Yet, for the mass gap problem or for a partial solution of the glueball spectrum, one should consider hyperfinite subalgebras. We show that a hyperfinite sector is constructible by fluctuations around a trivial topological field theory underlying Yang-Mills at large N . The hyperfiniteness problem has been suggested by the author as one of several problems arising as a byproduct of the Simons Center workshop "Mathematical Foundations of Quantum Field Theory", Jan 16-20 (2012).

1. Topological quantum field theory in large- N YM and hyperfiniteness

It is known¹ that in a $4d$ quantum field theory with a finite number of fields, under mild assumptions on the existence of the *KMS* states for any temperature, the von Neumann algebra of the observables is algebraically isomorphic to the unique type III_1 hyperfinite factor.

We recall that a von Neumann algebra is hyperfinite if it is the weak limit of a sequence of matrix algebras.

The situation gets more involved in the large- N limit of any field theory that carries fields in the adjoint representation of $SU(N)$, in particular in the large- N limit of pure $SU(N)$ Yang-Mills (YM) [1].

In the YM case the large- N limit can be properly defined in terms of the von Neumann algebra generated by Wilson loops, $\Psi(x, x; A)$, supported on a loop, L_{xx} , based at a point, x :

$$\Psi(x, x; A) = P \exp i \int_{L_{xx}} A_\alpha dx_\alpha \quad (1.1)$$

built by means of the YM connection, A_α . At leading large- N order the Wilson loops satisfy the Makeenko-Migdal loop equation [2, 3]:

$$\begin{aligned} \int_{L_{xx}} dx_\alpha &< \frac{N}{2g^2} \text{Tr} \left(\frac{\delta S_{YM}}{\delta A_\alpha(x)} \Psi(x, x; A) \right) > = \\ i \int_{L_{xx}} dx_\alpha \int_{L_{xx}} dy_\alpha \delta^{(4)}(x-y) &< \text{Tr} \Psi(x, y; A) > < \text{Tr} \Psi(y, x; A) > \end{aligned} \quad (1.2)$$

We can combine the expectation value $< \dots >$ with the normalized matrix trace $\frac{1}{N} \text{Tr}$ to define a new normalized trace $TR = < \frac{1}{N} \text{Tr}(\dots) >$ [4]. Then the problem is to find an operator solution, A_α , of the Makeenko-Migdal equation uniformly for all loops, with values in a certain operator algebra with normalizable trace $TR(1) = 1$. Such a solution is called the master field [5]. Such an algebra is of type II_1 because of the existence of the normalizable trace and it is explicitly known.

Indeed the ambient von Neumann algebra of the Makeenko-Migdal loop equation is the Cuntz algebra in its tracial representation with at least as many self-adjoint generators as the number of components of the gauge connection, i.e. 4 in $4d$ [6, 7, 8, 9, 10, 11].

We recall that the tracial representation of the Cuntz algebra is defined as follows [9, 10, 11]:

$$\begin{aligned} a_i a_j^* &= \delta_{ij} 1 \\ a_i |\Omega > &= 0 \\ \sum_i a_i^* a_i &= 1 - |\Omega > < \Omega| \\ TR(\dots) &= < \Omega | \dots | \Omega > \end{aligned} \quad (1.3)$$

and that the construction of the master field in terms of the Cuntz algebra [9, 10, 11] involves only four generators since, by a version of the Eguchi-Kawai reduction at large- N [13, 14, 15, 16, 17, 18, 19, 20], translations can be absorbed by gauge transformations [21, 22]:

$$SA_\mu S^{-1} = a_\mu^* + M_{\mu\nu} a_\nu + M_{\mu\nu\rho} a_\nu a_\rho + \dots \quad (1.4)$$

¹See pdf of Detlev Buchholz talk at the Simons Center workshop "Mathematical Foundations of Quantum Field Theory" Jan 16-20 (2012), hereby referred to as "this workshop".

However, the finite number of generators is only seemingly a simplification [23]. In fact, by Voiculescu work [24], the von Neumann algebra of the Cuntz algebra with more than one self-adjoint generator in its tracial representation is a type II_1 non-hyperfinite von Neumann algebra, that is algebraically isomorphic to a free group factor with the same number of generators, that is the main example of the elusive non-hyperfinite type II_1 factors. Therefore solving the Makeenko-Migdal equation is, to use just an euphemism, very difficult.

We should add that the von Neumann algebra generated by the actual solution need not to be non-hyperfinite (a string solution [25, 26, 27] ?) but there is no field theoretical reason why it should not.

Nevertheless, connected two-point correlation functions of local single trace gauge invariant operators, $O(x)$, of large- N YM or QCD are conjectured to be in a sense the most simple as possible, a sum of an infinite number of propagators of free fields (for simplicity of notation we consider the scalar case only) [28, 29, 30] at the leading non-trivial $\frac{1}{N}$ order:

$$\begin{aligned} G_O(p^2) &= \int e^{ipx} \langle O(x)O(0) \rangle_{conn} d^4x \\ &= \sum_k \frac{Z_k}{p^2 + m_k^2} \end{aligned} \quad (1.5)$$

saturating the logarithms of perturbation theory [28, 29, 30]:

$$G_O(p^2) \sim Z_O^2(p^2) p^{2L-4} \log\left(\frac{p^2}{\mu^2}\right) \quad (1.6)$$

where $Z_O(p^2)$ is a multiplicative renormalization related to the one-loop anomalous dimension, γ , of the operator, $O(x)$, whose RG -improved behavior is a fractional power of a logarithm:

$$Z_O(p^2) \sim \left[\log\left(\frac{p^2}{\mu^2}\right) \right]^{\frac{\gamma}{2\beta_0}} \quad (1.7)$$

with β_0 the first coefficient of the beta function [28, 29, 30]. It is an interesting problem², related to the actual computability of the glueballs spectrum, whether the corresponding local algebra is hyperfinite.

Hyperfiniteness can be interpreted as a condition on the number of local degrees of freedom of the theory³. Indeed hyperfiniteness is only slightly weaker than the existence of the KMS states for any temperature.

For example the bosonic string does not satisfy the KMS condition for all temperatures because of the Hagedorn transition [32], i.e. the divergence of the partition function at a certain finite temperature. However, the Hagedorn transition has been related to the tachyon of the bosonic string (see for example [33, 34]), in such a way that the glueball spectrum of large- N YM may or may not arise by a hyperfinite local algebra even in the likely case that a stringy description (obviously non-tachyonic) does exist.

²This problem has been proposed by the author at this workshop as one of the interesting open problems of quantum field theory.

³See pdf of Detlev Buchholz talk at this workshop.

Yet, if we are interested only in the problem of the mass gap [35], we may limit to correlators that involve only scalar states ⁴ and possibly to hyperfinite subalgebras [31, 30].

Therefore we may wonder as to whether there exist special Wilson loops that generate "small" subalgebras.

Surprisingly the answer is affirmative. There exist special Wilson loops, that we call twistor Wilson loops for geometrical reasons explained below, defined in pure $U(N)$ YM on non-commutative space-time, $R^2 \times R_\theta^2$, with complex coordinates, $(z, \bar{z}, \hat{u}, \hat{\bar{u}})$, built by a modified non-Hermitian connection:

$$\Psi(\hat{B}_\lambda; L_{ww}) = P \exp i \int_{L_{ww}} (\hat{A}_z + \lambda \hat{D}_u) dz + (\hat{A}_{\bar{z}} + \lambda^{-1} \hat{D}_{\bar{u}}) d\bar{z} \quad (1.8)$$

that define the observables of a trivial topological theory underlying the large- N limit of YM [31]:

$$\begin{aligned} < \frac{1}{\mathcal{N}} \text{Tr}_{\mathcal{N}} \Psi(\hat{B}_\lambda; L_{ww}) > = < \frac{1}{\mathcal{N}} \text{Tr}_{\mathcal{N}} \Psi(\hat{B}_1; L_{ww}) > \\ \lim_{\theta \rightarrow \infty} < \frac{1}{\mathcal{N}} \text{Tr}_{\mathcal{N}} \Psi(\hat{B}_\lambda; L_{ww}) > &= 1 \end{aligned} \quad (1.9)$$

with $\hat{D}_u = \partial_u + i\hat{A}_u$ the covariant derivative and $[u, \bar{u}] = \theta 1$, $[\partial_u, \partial_{\bar{u}}] = \theta^{-1} 1$. It is known that non-commutative YM in the limit of infinite non-commutativity, θ , coincides with the large- N limit of commutative YM , in such a way that the non-commutative theory realizes the same master field of the commutative one [19, 20, 36, 22].

The topological theory is trivial ⁵ because the generalized trace, TR , is exactly 1 for all the topological twistor Wilson loops for $\theta \rightarrow \infty$.

The trivial topological theory exists at all scales. Heuristically, in the same vein of the argument of the previous footnote, this may be related to the fact that at leading $\frac{1}{N}$ order the local algebra factorizes, i.e. it is in fact ultralocal. Indeed for every local single trace operator, $O_i(x_i)$, at leading $\frac{1}{N}$ order:

$$< O_1(x_1) \dots O_k(x_k) > = < O_1(x_1) > \dots < O_k(x_k) > \quad (1.10)$$

The triviality of twistor Wilson loops follows from the fact that in the large- θ limit they are gauge equivalent to ordinary Wilson loops supported on Lagrangian submanifolds of twistor space of complexified space-time:

$$\begin{aligned} &P \exp i \int_{L_{ww}} (A_z(z, \bar{z}, i\lambda z, i\lambda^{-1} \bar{z}) + i\lambda A_u(z, \bar{z}, i\lambda z, i\lambda^{-1} \bar{z})) dz \\ &+ (A_{\bar{z}}(z, \bar{z}, i\lambda z, i\lambda^{-1} \bar{z}) + i\lambda^{-1} A_{\bar{u}}(z, \bar{z}, i\lambda z, i\lambda^{-1} \bar{z})) d\bar{z} \end{aligned} \quad (1.11)$$

with the parameter λ playing the role of the (Lagrangian) fiber of the twistor fibration [31]. In the language of local wedge algebras ⁶ these loops are supported on (the analytic continuation

⁴It is believed that the lowest glueball state is a scalar.

⁵The existence of a possibly trivial topological theory in the infrared underlying every theory with a mass gap, in particular pure YM , has been advocated by Edward Witten in his talk at this workshop. Such a trivial topological theory underlying large- N YM in $4d$ had been constructed in [37, 38, 39, 31, 30].

⁶See pdf of Detlev Buchholz talk at this conference.

of) Lagrangian wedges. Remarkably the support property implies triviality [31], because of the vanishing of the coefficients of the effective propagators on the support:

$$\begin{aligned} \dot{z}\dot{\bar{z}} + i^2\lambda\dot{z}\lambda^{-1}\dot{\bar{z}} &= 0 \\ \dot{z}\bar{z} + i^2\lambda\dot{z}\lambda^{-1}\bar{z} &= 0 \\ z\dot{\bar{z}} + i^2\lambda z\lambda^{-1}\dot{\bar{z}} &= 0 \end{aligned} \quad (1.12)$$

and, via triviality, implies the following localization property, of the trivial topological algebra of twistor Wilson loops, in function space [37, 31].

There exists a change of variables in the YM functional integral such that a new loop equation [37, 31] in the trivial topological sector:

$$\int Tr \frac{\delta}{\delta \mu'(z, \bar{z})} (e^{-\Gamma \Psi(B'; L_{zz})}) \delta \mu' = 0 \quad (1.13)$$

reads:

$$< Tr(\frac{\delta \Gamma}{\delta \mu'(z, \bar{z})} \Psi(B'; L_{zz})) > = \frac{1}{\pi} \int_{L_{zz}} \frac{dw}{z-w} < Tr \Psi(B'; L_{zw}) > < Tr \Psi(B'; L_{wz}) > \quad (1.14)$$

The change of variables involves a non- $SUSY$ version [37, 31] of the Nicolai map [40, 41, 42, 43]:

$$\begin{aligned} Z &= \int \exp\left(-\frac{16\pi^2 N Q}{2g^2} - \frac{N}{4g^2} \int tr_f (F_{\alpha\beta}^-)^2 d^4x\right) \delta A \\ 1 &= \int \delta(F_{\alpha\beta}^- - \mu_{\alpha\beta}^-) \delta \mu_{\alpha\beta}^- \\ Z &= \int \exp\left(-\frac{16\pi^2 N Q}{g^2} - \frac{N}{4g^2} \int tr_f (\mu_{\alpha\beta}^-)^2 d^4x\right) \delta(F_{\alpha\beta}^- - \mu_{\alpha\beta}^-) \delta A \delta \mu_{\alpha\beta}^- \end{aligned} \quad (1.15)$$

and the holomorphic gauge, $B_{\bar{z}} = 0$ [37, 31]:

$$\begin{aligned} < \dots > = Z^{-1} \int \delta n \delta \bar{n} \int_{C_1} \delta \mu' \dots \exp\left(-\frac{N 8 \pi^2}{g^2} Q - \frac{N 4}{g^2} \int Tr_f (\mu \bar{\mu}) + Tr_f (n + \bar{n})^2 d^4x\right) \\ &\quad \delta(-iF_B - \mu - \theta^{-1}1) \delta(-i\partial_A \bar{D} - n) \delta(-i\bar{\partial}_A D - \bar{n}) \frac{\delta \mu}{\delta \mu'} \delta A \delta \bar{A} \delta D \delta \bar{D} \end{aligned} \quad (1.16)$$

The new holomorphic loop equation [37, 31] can be regularized in a gauge-invariant way by analytic continuation to Minkowski:

$$\begin{aligned} < Tr(\frac{\delta \Gamma_M}{\delta \mu'(z_+, z_-)} \Psi(B'; L_{z_+ z_-})) > = \\ \frac{1}{\pi} \int_{L_{z_+ z_-}} \left(P \frac{dw_+}{z_+ - w_+} - idw_+ \pi \delta(z_+ - w_+)\right) < Tr \Psi(B'; L_{z_+ w_+}) > < Tr \Psi(B'; L_{w_+ z_+}) > \end{aligned} \quad (1.17)$$

Finally, deforming the standard Makeenko-Migdal loop with the shape of the symbol ∞ , to a cusped loop with zero cusp angle at the non-trivial self-intersection, the localization property follows [37, 31]:

$$< Tr(\frac{\delta \Gamma}{\delta \mu'(z_+, z_-)} \Psi(B'; L_{z_+ z_-})) > = 0 \quad (1.18)$$

This equation expresses the fact that matrix elements of the equation of motion of Γ vanish, when restricted to the sub-algebra of twistor Wilson loops:

$$\langle \text{Tr}(\Psi(B; L'_{z_+z_-})) \frac{\delta \Gamma}{\delta \mu(z_+, z_-)} \Psi(B; L''_{z_+z_-}) \rangle = 0 \quad (1.19)$$

There is a homological interpretation of the localization such that the holomorphic loop equation is localized by the addition to the loop of vanishing boundaries, i.e. backtracking arcs ending with cusps [37, 31], in the dual way to the localization of cohomology classes, that involves deforming by coboundaries [44, 45, 46, 47, 48, 49, 55].

Thus the topological sector is localized at the critical points of the effective action, Γ .

Γ contains the interesting information of the localization, since it is naturally defined on the physical wedge, rather than on the topological one.

Indeed, though the topological theory is trivial at large- N , the effective action, Γ , admits at subleading $1/N$ order, non-trivial fluctuations around the critical points, supported on a "transverse" Lagrangian wedge, that is a physical wedge:

$$(z, \bar{z}, i\lambda z, i\lambda^{-1} \bar{z}) \rightarrow (z_+, z_-, -\lambda z_+, -\lambda^{-1} z_-) \quad (1.20)$$

i.e. a certain Lagrangian wedge that is defined through the analytic continuation, as operators, of the topological twistor Wilson loops to the physical twistor Wilson loops:

$$\Psi(\hat{B}_\lambda; L_{ww}) \rightarrow P \exp i \int_{L_{ww}} (\hat{A}_{z_+} + i\lambda \hat{D}_{u_+}) dz_+ + (\hat{A}_{z_-} + i\lambda^{-1} \hat{D}_{u_-}) dz_- \quad (1.21)$$

The twistor Wilson loops supported on the physical wedge satisfy the same Minkowski loop equation, Eq.(1.17), but not the localization property, Eq.(1.18), since their v.e.v. is non-trivial. In particular their v.e.v. does not stay uniformly bounded from zero and from infinity deforming to a cusped loop, because of the cusp anomaly [50, 51, 52].

The algebra of observables of the topological theory can be realized explicitly as the closure of a dense set in function space that involves a lattice of surface operators supported on Lagrangian submanifolds [37, 31]. Mathematically these are local systems associated to the topological theory, obtained interpreting [31] the Nicolai map as hyper-Kahler reduction [53, 54] on a lattice divisor:

$$F_{\alpha\beta}^- = \sum_p \mu_{\alpha\beta}^-(p) \delta^{(2)}(z - z_p) \quad (1.22)$$

On the lattice hyper-Kahler quotient the physical fluctuations around the topological theory are locally abelian in function space, all the other non-abelian degrees of freedom being zero modes of the Jacobian of the Nicolai map associated to the moduli of the local system [31].

The theory becomes locally abelian [31] because of the automatic commutativity [56, 57, 58, 59] of the triple, $\mu_{\alpha\beta}^-(p)$, at each lattice point, p , of the "lattice field of residues", due to the singular nature of the Hitchin equations, Eq.(1.22) [60, 61, 62].

Therefore the physical fluctuations around the topological theory become computable in the large- N limit.

On the lattice hyper-Kahler quotient the critical points of the effective action can be found as fixed points for the action, on the Nicolai variables in function space, of the semigroup that rescales

the fiber of the Lagrangian twistor fibration, because of the λ -independence of the v.e.v. of twistor Wilson loops:

$$\langle \frac{1}{\mathcal{N}} \text{Tr}_{\mathcal{N}} \Psi(B_{\lambda}; L_{ww}) \rangle = \langle \frac{1}{\mathcal{N}} \text{Tr}_{\mathcal{N}} \Psi(B_1; L_{ww}) \rangle \quad (1.23)$$

A large- N beta function of Novikov-Shifman-Vainshtein-Zakharov type [63] at the leading $1/N$ order follows [37, 31].

At the next to leading $1/N$ order, the mass gap, the glueballs spectrum, the anomalous dimensions [64, 65] and the hyperfiniteness follow as well, for fluctuations supported on the transverse Lagrangian wedge [31, 30].

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